

Section D

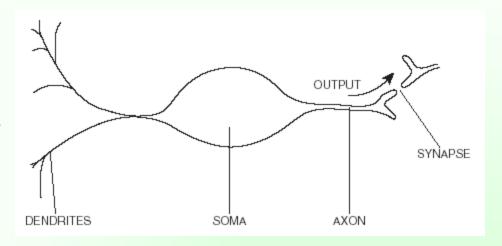
Neural Networks



- Artificial neural networks (ANNs) provide a practical method for learning
 - real-valued functions
 - discrete-valued functions
 - vector-valued functions
- Robust to errors in training data
 - Successfully applied to such problems as
 - interpreting visual scenes
 - speech recognition
 - learning robot control strategies

Biological Neurons

The human brain is made up of billions of simple processing units – neurons.



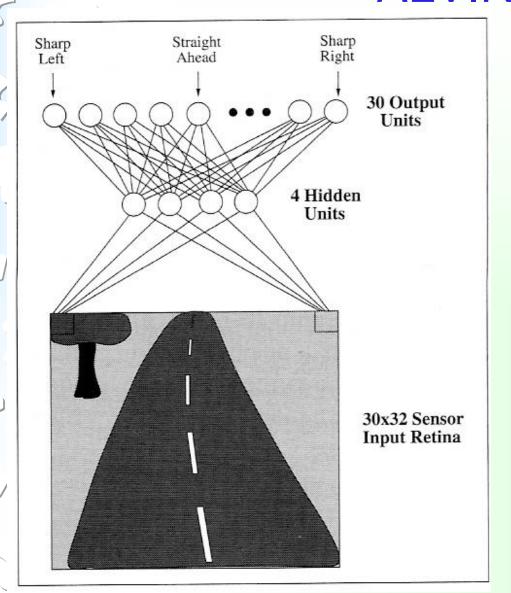
Inputs are received on dendrites, and if the input levels are over a threshold, the neuron fires, passing a signal through the axon to the synapse which then connects to another neuron.

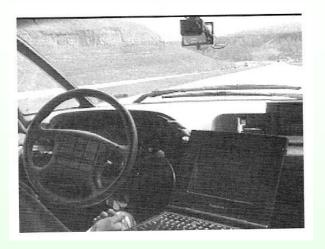


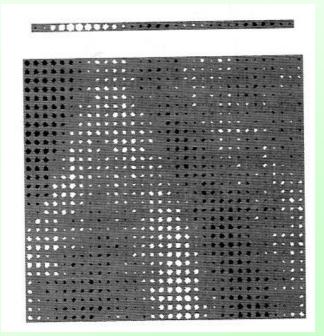
ALVINN uses a learned ANN to steer an autonomous vehicle driving at normal speeds on public highways

- Input to network: 30x32 grid of pixel intensities obtained from a forward-pointed camera mounted on the vehicle
- Output: direction in which the vehicle is steered
- Trained to mimic observed steering commands of a human driving the vehicle for approximately 5 minutes

ALVINN







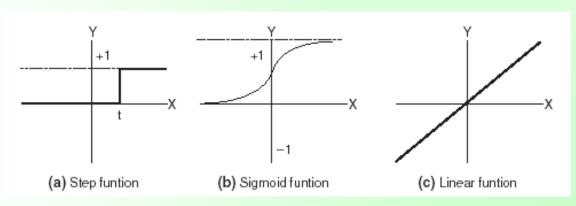


- ANN learning well-suit to problems which the training data corresponds to noisy, complex data (inputs from cameras or microphones)
- Can also be used for problems with symbolic representations
- Most appropriate for problems where
 - Instances have many attribute-value pairs
 - Target function output may be discrete-valued, real-valued, or a vector of several real- or discrete-valued attributes
 - Training examples may contain errors
 - Long training times are acceptable
 - Fast evaluation of the learned target function may be required
 - The ability for humans to understand the learned target function is not important



- Artificial neurons are based on biological neurons.
- Each neuron in the network receives one or more inputs.
 - An activation function is applied to the inputs, which determines the output of the neuron the activation level.

The charts on the right show three typical activation functions.



Artificial Neurons (2)

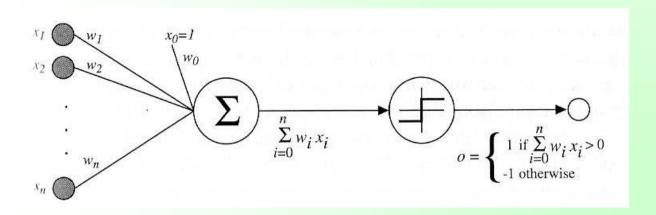
• A typical activation function works as follows:

$$X = \sum_{i=1}^{n} w_i x_i \qquad Y = \begin{cases} +1 & \text{for } X > t \\ 0 & \text{for } X \le t \end{cases}$$

- Each node i has a weight, w_i associated with it. The input to node i is x_i .
- t/is/the threshold.
- So if the weighted sum of the inputs to the neuron is above the threshold, then the neuron fires.

Perceptrons

- A perceptron is a single neuron that classifies a set of inputs into one of two categories (usually 1 or 1).
- If the inputs are in the form of a grid, a perceptron can be used to recognize visual images of shapes.
- The perceptron usually uses a step function, which returns 1 if the weighted sum of inputs exceeds a threshold, and 0 otherwise.





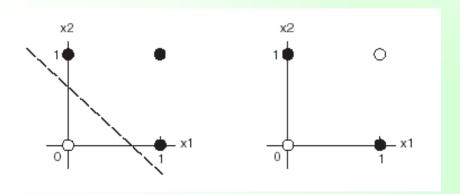
- Learning involves choosing values for the weights
- The perceptron is trained as follows:
 - First, inputs are given random weights (usually between -0.5 and 0.5).
 - An item of training data is presented. If the perceptron mis-classifies it, the weights are modified according to the following:

$$w_i \leftarrow w_i + (a \times x_i \times (t - o))$$

- where *t* is the target output for the training example, *o* is the output generated by the preceptron and *a* is the learning rate, between 0 and 1 (usually small such as 0.1)
- Cycle through training examples until successfully classify all examples
 - Each cycle known as an **epoch**



- Perceptrons can only classify linearly separable functions.
- The first of the following graphs shows a linearly separable function (OR).
- The second is not linearly separable (Exclusive-OR).

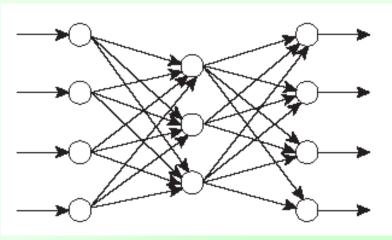




- Perceptron training rule only converges when training examples are linearly separable and a has a small learning constant
- Another approach uses the *delta rule* and gradient descent
 - Same basic rule for finding update value
 - Changes
 - Do not incorporate the threshold in the output value (unthresholded perceptron)
 - Wait to update weight until cycle is complete
 - Converges asymptotically toward the minimum error hypothesis, possibly requiring unbounded time, but converges regardless of whether the training data are linearly separable

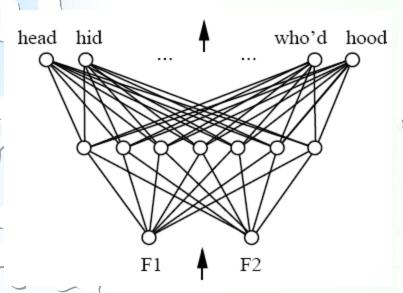
Multilayer Neural Networks

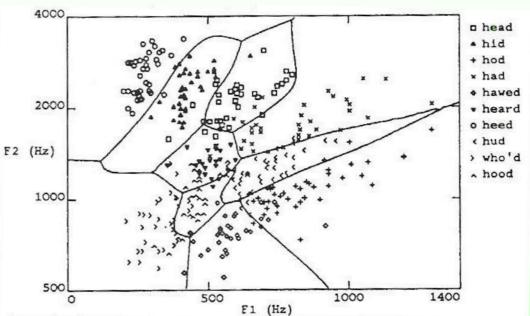
- Multilayer neural networks can classify a range of functions, including non linearly separable ones.
- Each input layer neuron connects to all neurons in the hidden layer.
 - The neurons in the hidden layer connect to all neurons in the output layer.



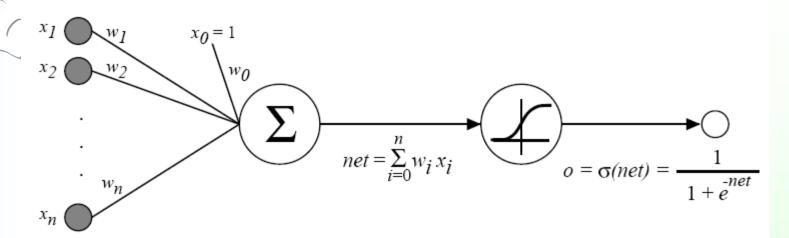
A feed-forward network

Speech Recognition ANN





Sigmoid Unit



 $\sigma(x)$ is the sigmoid function

$$\frac{1}{1+e^{-x}}$$

Nice property: differentiable

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x))$$

Derive gradient descent rules to train

- One sigmoid unit node
- Multilayer networks of sigmoid units



Backpropagation

- Multilayer neural networks learn in the same way as perceptrons.
- However, there are many more weights, and it is important to assign credit (or blame) correctly when changing weights.
- E sums the errors over all of the network output units

$$E(\vec{w}) \equiv \frac{1}{2} \sum_{d \in D} \sum_{k \in outputs} (t_{kd} - o_{kd})^2$$



Backpropagation Algorithm

Create a feed-forward network with n_{in} inputs, n_{hidden} hidden units, and n_{out} output units.

Initialize all network weights to small random numbers Until termination condition is met, Do

- For each $\langle x,t \rangle$ in training examples, Do *Propagate the input forward through the network:*
 - 1. Input the instance x to the network and compute the output o_u of every unit u in the network

Propagate the errors backward through the network:

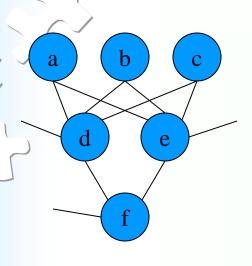
- 2. For each network output unit k, calculate its error term δ_k $\delta_k \leftarrow o_k (1 o_k)(t_k o_k)$
- 3. For each hidden unit h, calculate its error term δ_h $\delta_h \leftarrow o_h(1-o_h) \sum_{k \in \text{outputs}} w_{kh} \delta_k$
- 4. Update each network weight w_{ji}

$$W_{ji} \leftarrow W_{ji} + \Delta W_{ji}$$

where

$$\Delta w_{ji} = \alpha \delta_j x_{ji}$$

Example: Learning AND



Training Data: AND(1,0,1) = 0AND(1,1,1) = 1

Alpha =
$$0.1$$

Initial Weights:

$$w_da = .2$$

$$w_db = .1$$

$$w_dc = -.1$$

$$w_d0 = .1$$

$$w_ea = -.5$$

$$w_eb = .3$$

$$w_ec = -.2$$

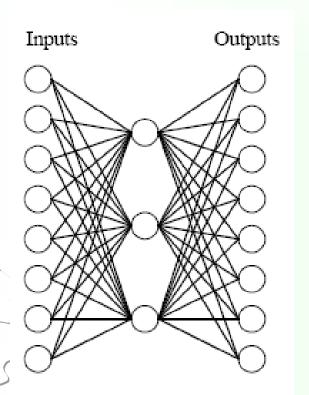
$$w_e0 = 0$$

$$w_fd = .4$$

$$w_fe = -.2$$

$$w_f0 = -.1$$

Hidden Layer representation



Target Function:

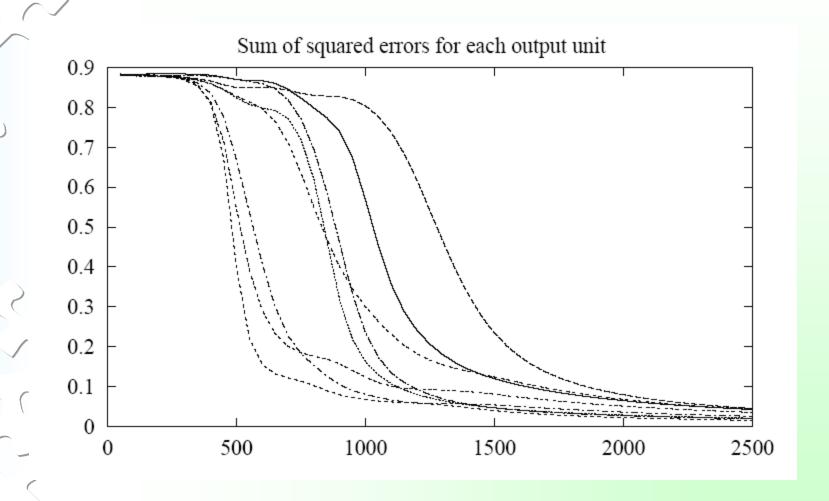
Input		Output
10000000	\rightarrow	10000000
01000000	\rightarrow	01000000
00100000	\rightarrow	00100000
00010000	\rightarrow	00010000
00001000	\rightarrow	00001000
00000100	\rightarrow	00000100
00000010	\rightarrow	00000010
00000001	\rightarrow	00000001

Can this be learned?

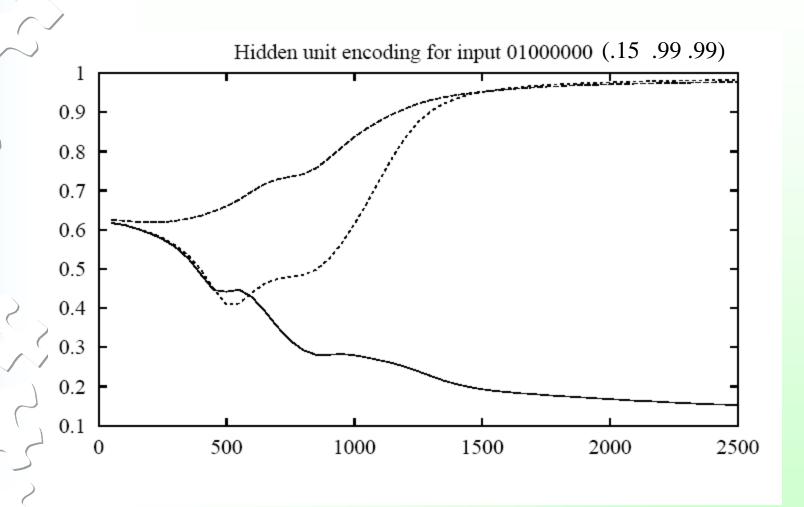
Yes

	Input	Hidden	Output
		Values	
	10000000	\rightarrow .89 .04 .08	→ 10000000
	01000000	→ .15 .99 .99	→ 01000000
	00100000	→ .01 .97 .27	→ 00100000
	00010000	→ .99 .97 .71	→ 00010000
	00001000	\rightarrow .03 .05 .02	→ 00001000
	00000100	→ .01 .11 .88	→ 00000100
	00000010	→ .80 .01 .98	→ 00000010
,	00000001	→ .60 .94 .01	→ 00000001

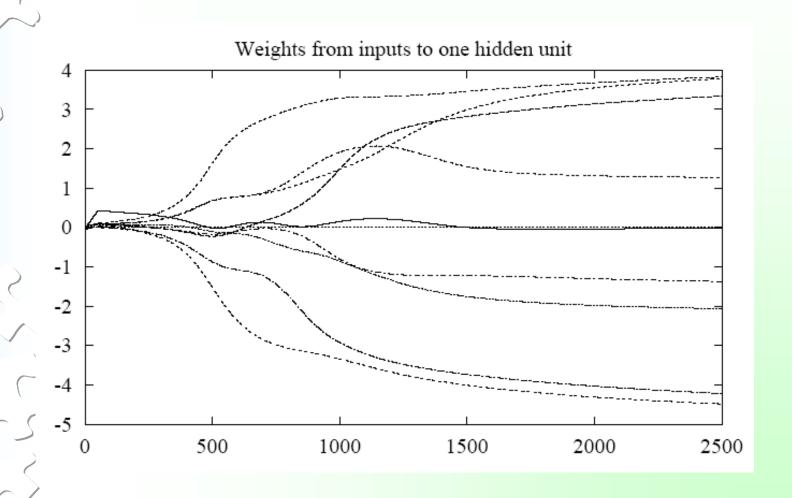
Plots of Squared Error



Hidden Unit



Evolving weights



Momentum

- One of many variations
- Modify the update rule by making the weight update on the *n*th iteration depend partially on the update that occurred in the (*n*-1)th iteration

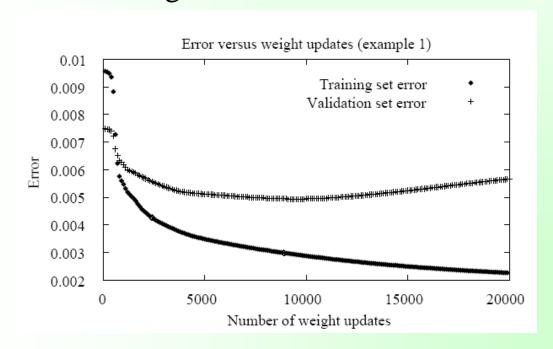
$$\Delta w_{ji}(n) = \alpha \delta_j x_{ji} + \beta \Delta w_{ji}(n-1)$$

- Minimizes error over training examples
- Speeds up training since it can take 1000s of iterations



Continue until error falls below some predefined threshold

- Bad choice because Backpropagation is susceptible to overfitting
- Won't be able to generalize as well over unseen data

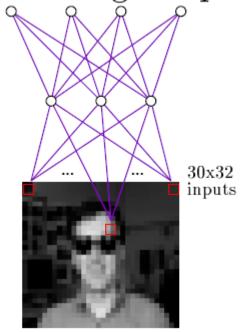


Cross Validation

- Common approach to avoid overfitting
- Reserve part of the training data for testing
- m examples are partitioned into k disjoint subsets
- Run the procedure k times
 - Each time a different one of these subsets is used as validation
- Determine the number of iterations that yield the best performance
- Mean of the number of iterations is used to train all n examples

Neural Nets for Face Recognition

left strt rght up











Typical input images

Hidden Unit Weights

